DUSO Mathematics League 2016 - 2017

CONTEST #3.

SOLUTIONS

3 - 1. $\boxed{\textbf{33.6}}$ Alex earns \$30 per hour, Bridget earns \$20 per hour, and Carl earns \$10 per hour. Together, the students earn \$60 per hour (or \$1 per minute). Therefore, they would have to work 2016 minutes to earn 2016 dollars. The number of hours is thus $2016 \div 60 = \frac{168}{5}$ or 33.6.

3 - 2. $\boxed{\textbf{2}}$ Notice that $12^c = (8^b)^c = (4^a)^{bc} = 16$. By laws of exponents, this implies that $4^{abc} = 16$, so $abc = \mathbf{2}$.

---

3 - 3. $\boxed{\textbf{135}}$ We solve $4x + x + x = 180$ to obtain $m\angle ACB = 30°$ and $m\angle CAB = 120°$. We also have $m\angle ABD = 180° - 30° = 150°$, so $m\angle BAD = \frac{30}{2} = 15°$, so our answer is $120 + 15 = \mathbf{135}°$.

3 - 4. $\boxed{\dfrac{\mathbf{51}}{\mathbf{5}}}$ The slope of ℓ is the negative reciprocal of the slope of $\overline{PP'}$. The slope of $\overline{PP'}$ is $\dfrac{10-6}{1-3} = -2$, so the slope of ℓ is $\dfrac{1}{2}$. The midpoint $(2, 8)$ of $\overline{PP'}$ is on ℓ, so line ℓ has equation $y - 8 = \dfrac{1}{2}(x-2)$ or $y = \dfrac{1}{2}x + 7$. The slope of $\overline{QQ'}$ is $\dfrac{b-9}{a-2} = \dfrac{-2}{1}$, so $b = -2a + 13$. The midpoint of $\overline{QQ'}$ is on $y = \dfrac{1}{2}x + 7$, so $\left(\dfrac{a+2}{2}, -a+11\right)$ satisfies $y = \dfrac{1}{2}x + 7$. Substituting, $-a + 11 = \dfrac{1}{2}\left(\dfrac{a+2}{2}\right) + 7$, which solves to obtain $a = \dfrac{14}{5}$ and $b = -2 \cdot \dfrac{14}{5} + 13 = \dfrac{37}{5}$, so the sum $a + b$ is $\dfrac{\mathbf{51}}{\mathbf{5}}$.

---

3 - 5. $\boxed{\textbf{110684}}$ Notice that $f(x)$ is even. That is, all of the powers of x are even, so $f(-a) = f(a)$ for all a. Thus, $f(-7) = f(7) = \mathbf{110684}$.

3 - 6. $\boxed{\dfrac{\mathbf{171}}{\mathbf{221}}}$ The value $\sin G = \sin(180° - T - R)$, or $\sin G = \sin(T + R)$. By formula, $\sin G = \sin T \cos R + \cos T \sin R = \frac{5}{13} \cdot \frac{15}{17} + \frac{12}{13} \cdot \frac{8}{17}$, or $\dfrac{\mathbf{171}}{\mathbf{221}}$.

R-1. A triangle has sides of length 80, 150, and 170. Compute the degree measure of the largest angle of the triangle.
R-1Sol. $\boxed{\mathbf{90}}$ The sides satisfy the Pythagorean Theorem, so the triangle is a right triangle, and the largest angle measures $\mathbf{90}^\circ$.

R-2. *Let N be the number you will receive.* In parallelogram $DUSO$, the measures of angles D and U differ by N°. If $\angle S$ is obtuse, compute the $m\angle O$.
R-2Sol. $\boxed{\mathbf{45}}$ The question implies two linear equations: $D - U = N$ by the problem statement and $D + U = 180$ by properties of parallelograms. Notice that $D = S$ and $U = O$, so adding and substituting yields $S = \dfrac{180 + N}{2}$ and $O = 180 - \dfrac{180 + N}{2} \to 90 - \dfrac{N}{2}$. Substitution yields $m\angle O = 90 - 45 = \mathbf{45}$.

R-3. *Let N be the number you will receive.* Jimmy, Timmy, and Kimmy are three teenagers whose ages add to N. Jimmy's age is one year greater than the average of the three ages. Timmy is three years older than Kimmy. Compute Kimmy's age.
R-3Sol. $\boxed{\mathbf{13}}$ Jimmy's age is $\dfrac{N}{3} + 1$. Timmy's age is $K + 3$ if Kimmy's age is K, so solve $K + K + 3 + \dfrac{N}{3} + 1 = N$. This implies $2K + 4 = \dfrac{2N}{3}$, or $2K = \dfrac{2N - 12}{3}$, so $K = \dfrac{2N - 12}{6}$. Substitution yields $K = \dfrac{2 \cdot 45 - 12}{6} = \mathbf{13}$.

R-4. *Let N be the number you will receive.* In a room with N people, every person is wearing either a red shirt or a blue shirt. Each person in a red shirt shakes hands with every person in a blue shirt exactly once. If a total of 30 handshakes take place, and there are more people wearing red shirts than blue shirts, how many people are wearing red shirts?
R-4Sol. $\boxed{\mathbf{10}}$ Find a factor pair of 30 that adds to N. Since $N = 13$, that pair must be 3 and 10. There are **10** people wearing red shirts.

R-5. *Let N be the number you will receive.* In a theater with N rows of seats, each row has three more seats than the previous row. There are 205 seats in the theater. Compute the number of seats in the row with the fewest seats.
R-5Sol. $\boxed{\mathbf{7}}$ This is an arithmetic series of first term x and common difference 3, so $\dfrac{N}{2}(x + (x + 3(N - 1))) = 205$. Substituting and solving obtains $x = \mathbf{7}$.

Author: George Reuter - coachreu@gmail.com – Reviewer: Michael Curry - currymath@gmail.com